

# Scaling Neural Program Synthesis with Distribution-based Search

Nathanaël  
Fijalkow<sup>1,2</sup>

Guillaume  
Lagarde<sup>1</sup>

**Théo  
Matricon**<sup>1</sup>

Kevin Ellis<sup>3</sup>

Pierre  
Ohlmann<sup>4</sup>

Akarsh  
Potta<sup>5</sup>

<sup>1</sup>CNRS, LaBRI and  
Université de Bordeaux,  
France

<sup>2</sup>The Alan Turing  
Institute of data science,  
United Kingdom

<sup>3</sup>Cornell University,  
United States

<sup>4</sup>University of Paris,  
France

<sup>5</sup>Indian Institute of  
Technology Bombay,  
India

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$f : \text{list}(\text{int}) \rightarrow \text{list}(\text{int})$   
 $f \text{ var0} = \text{map } (\lambda x. \text{mod } x \ 7) (\text{map } (+ \ 3) \text{ var0})$

## SETTING

### DSL

```
sort : list(int) → list(int)
car : list(t) → t
cdr : list(t) → list(t)
map : (t1 → t2) →
      list(t1) → list(t2)
range : int → list(int)
+ : int → int → int
1 : int
⋮
```

### Syntactic constraints

```
program type list(int) → int
program depth ≤ 6
type size ≤ 10
type nesting ≤ 4
variable depth ≥ 3
no two consecutive sort
⋮
```

## PREPROCESSING

### Prediction model

Training phase

### CFG

```
S → I, 0
⋮
I, d → one
I, d → car1(L[I], d+1)
I, d → plus(I, d+1; I, d+1)
⋮
L[I], d → range(I, d+1)
L[I], d → var #d ≥ 3
L[I], d → sort(L[I], d+1)
L[I], d → cdr1(L[I], d+1)
L[I], d → car1(L[L[I]], d+1)
L[I], d → map1,1(ItoI, d+1;
                  L[I], d+1)
⋮
ItoI, d → car1tot(L[ItoI], d+1)
ItoI, d → plus(I, d+1)
⋮
```

## QUERY

Weights prediction

weights

### PCFG

```
S  $\xrightarrow{w}$  I, 0
⋮
I, d  $\xrightarrow{w}$  one
I, d  $\xrightarrow{w}$  car1(L[I], d+1)
I, d  $\xrightarrow{w}$  plus(I, d+1; I, d+1)
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L[I], d  $\xrightarrow{w}$  map1,1(ItoI, d+1;
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ItoI, d  $\xrightarrow{w}$  car1tot(L[ItoI], d+1)
ItoI, d  $\xrightarrow{w}$  plus(I, d+1)
⋮
```

USER

I/O

solution program

Search algorithm

Enumerate likely programs until meeting I/O specification

Pipeline for neural predictions for syntax guided program synthesis.

NN predicts a PCFG  $\rightarrow$  induces a distribution  $\mathcal{D}$  over programs

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Goal: find  $P$  as quickly as possible

Evaluation criterion for an algorithm  $A$  given  $\mathcal{D}$ ?

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Trade-off: Quality vs Quantity

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## Theorem

*The HEAP SEARCH algorithm is loss optimal: it enumerates every program exactly once and in non-increasing order of probabilities.*

It uses a data structure made of heaps and hashing tables to efficiently enumerate programs.

Sampling Algorithms: may generate a program multiple times but do not require memory.

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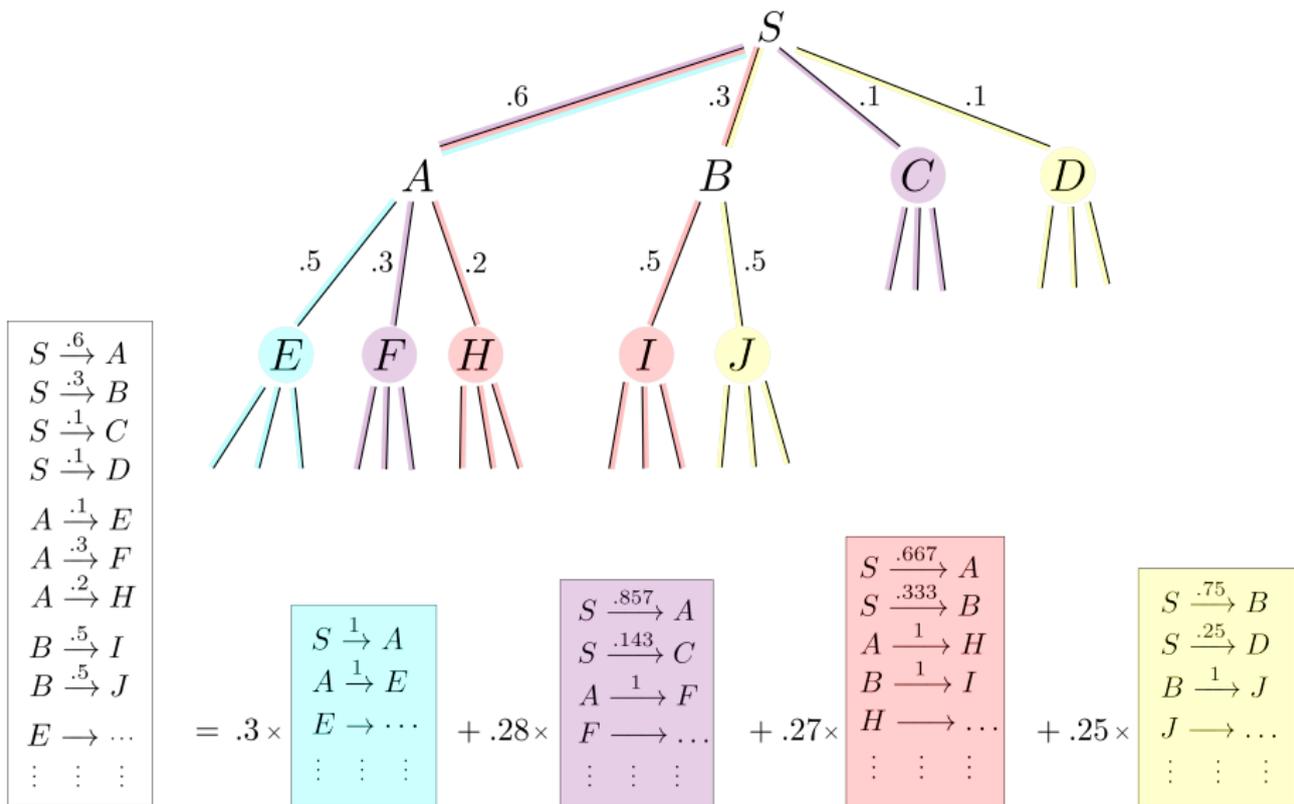
### Theorem

*The Sqrt SAMPLING algorithm is loss optimal among sampling algorithms.*

Sqrt SAMPLING samples program from the square root distribution of the prior distribution  $\mathcal{D}$ .

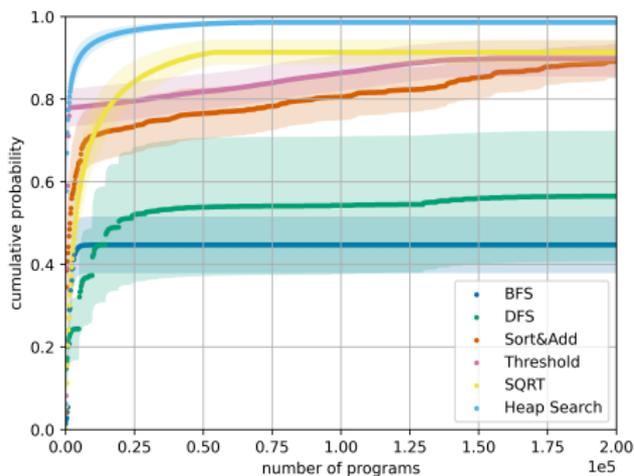
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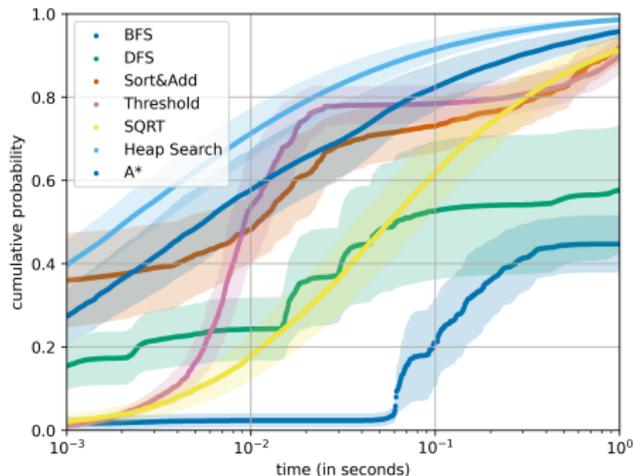


The grammar splitter: a balanced partition with imbalance  $\alpha = \frac{.3}{.25} = 1.2$ .

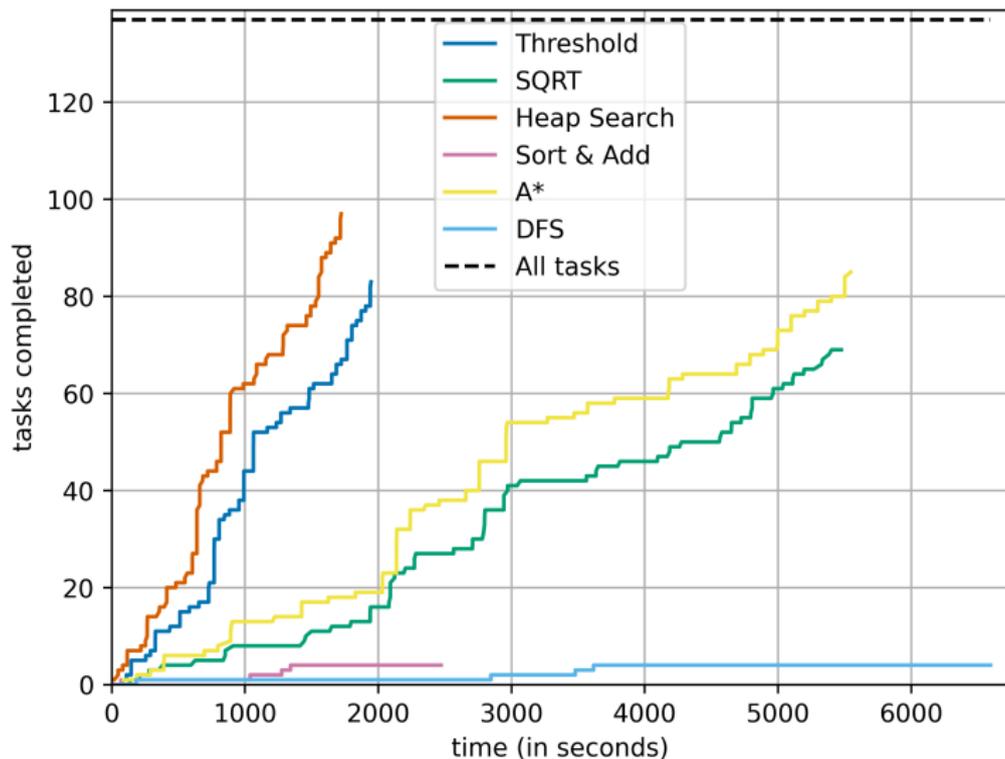
# Experiments



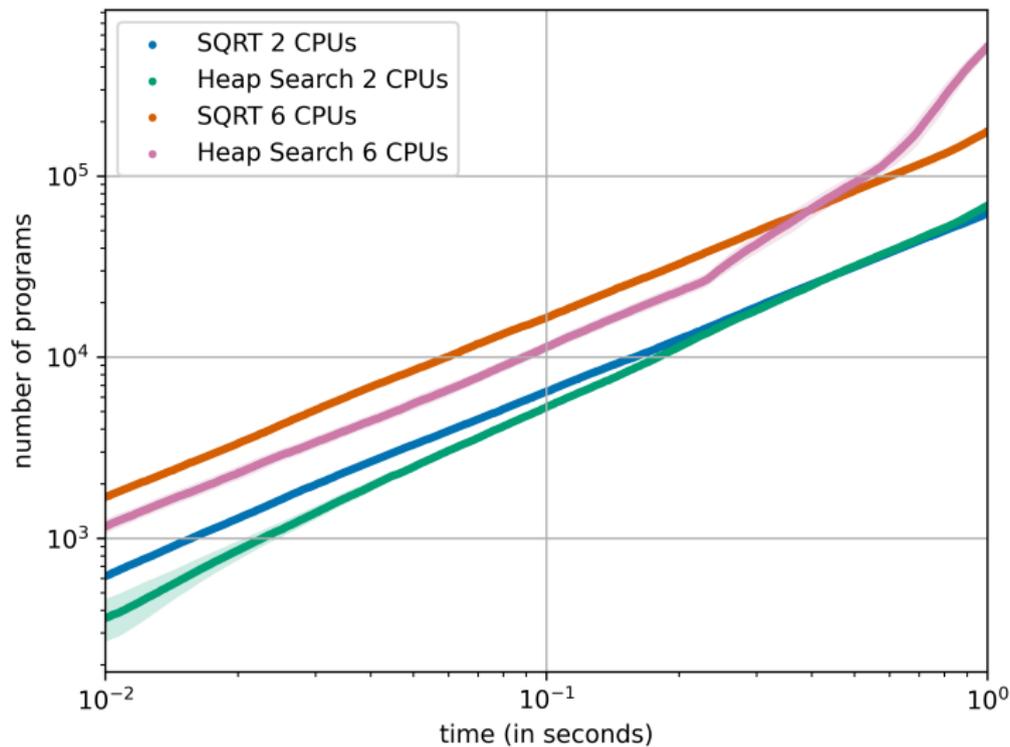
Cumulative probability against number of program output



Cumulative probability against time in log-scale



Comparing all search algorithms on the DreamCoder reduced dataset with machine-learned PCFGs



Parallel implementations of HEAP SEARCH and SQRT SAMPLING using the grammar splitter

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Code: [github.com/nathanael-fijalkow/DeepSynth](https://github.com/nathanael-fijalkow/DeepSynth)